DOCUMENT RESUME

ED 295 827 SE 049 208

TITLE	Michigan Essential Goals and Objectives for Mathematics Education.
INSTITUTION	Michigan State Board of Education, Lansing.
PUB DATE	Mar 88
NOTE	44p.; Graphs and pages with gray shaded areas may not reproduce well.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Algebra; Calculators; *Curriculum Development; *Elementary School Mathematics; Elementary Secondary Education; Geometry; *Mathematics Education; *Mathematics Instruction; Measurement; Number Concepts; Problem Solving; *Secondary School Mathematics; State Curriculum Guides; State Programs
IDENTIFIERS	*Michigan

ABSTRACT

This document is designed to assist administrators and teachers in planning, developing, and implementing grades K-9 mathematics programs and to provide some guidelines for grades 10-12 instruction. It is intended to provide a philosophical foundation and curricular framework from which educators may construct a comprehensive local program to meet the instructional needs of students. The document illustrates the integration of mathematical content with process skills. The mathematical content strands are: (1) whole numbers and numeration; (2) fractions, decimals, ratio, and percent; (3) measurement; (4) geometry; (5) statistics and probability; (6) algebraic ideas; (7) problem solving and logical reasoning; and (8) calculators. (PK)

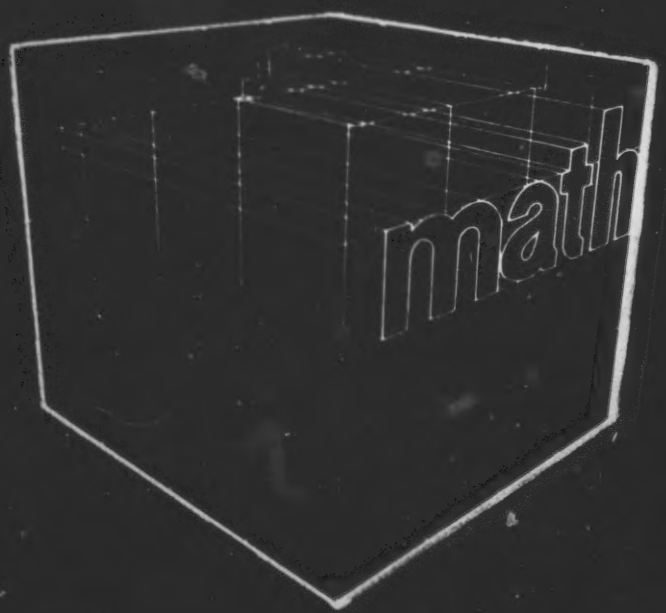
****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED295827

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☒ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Pearl C. Cullison
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

MICHIGAN ESSENTIAL GOALS AND OBJECTIVES FOR MATHEMATICS EDUCATION



SE 049 208

MICHIGAN STATE BOARD OF EDUCATION

Michigan
State Board of Education

Barbara Roberts Mason
President

Dorothy Beardmore
Vice President

Cherry Jacobus
Secretary

Dr. Gumecindo Salas
Treasurer

Dr. Edmund F. Vandette
NASBE Delegate

Carroll M. Hutton

Annetta Miller

Norman Otto Stockmeyer, Sr.

Ex-Officio Members

James J. Blanchard
Governor

Gary D. Hawks
Interim Superintendent of Public Instruction

FOREWORD

The Michigan Department of Education is pleased to present the revised "Michigan Essential Goals and Objectives for Mathematics Education." This document sets forth the essential mathematical learning for elementary and middle schools.

The "Michigan Essential Goals and Objectives for Mathematics Education" outlines the curriculum content necessary to foster understanding and promote reasoning, thinking, problem-solving, and application skills. Individual schools may modify and expand the content, as appropriate, for their particular student populations. This document outlines those essential concepts and skills appropriate for learners in kindergarten through grade nine. While sequences are suggested, teachers' judgment about a particular student's readiness for more advanced instruction will ultimately determine when new concepts and skills are introduced. The organization of contents and processes in this document is intended to help educators move each student from skill acquisition to higher-order learning while, at all times, fully engaging the students in rigorous academic study.

The "Michigan Essential Goals and Objectives for Mathematics Education" should help elementary and middle/junior high school educators shape a mathematics curriculum that prepares students for the challenges of secondary school and beyond. The building blocks of an academic foundation are important not only to children and their family but also to the future of our communities and the nation as a whole.

It is fitting to thank the many individuals, associations, and agencies who have provided assistance and support in the development and review of this document.

Gary D. Hawks
Interim Superintendent of
Public Instruction

March, 1988

STATEMENT OF PURPOSE

These essential goals and objectives are designed to assist administrators and teachers in planning, developing and implementing grades K-9 mathematics programs and to provide some guidelines for grades 10-12 instruction. They provide a philosophical foundation and curricular framework from which educators may construct a comprehensive local program to meet the instructional needs of their students.

This document illustrates the integration of mathematical content with process skills. In addition, it upgrades the bench mark expectations for achievement to be commensurate with the demands of a technological society.

Table of Contents

	Page
Introduction	2
Framework	4
New Directions	5
Mathematics Content Strands	
1. Whole Numbers and Numeration	7
2. Fractions, Decimals, Ratio and Percent	12
3. Measurement	20
4. Geometry	23
5. Statistics and Probability	26
6. Algebraic Ideas	29
7. Problem Solving and Logical Reasoning	33
8. Calculators	35

Michigan Essential Goals and Objectives for Mathematics Education

Introduction

Quantitative thought and understanding form the keystone for the new mathematics objectives for Michigan. Support for this comes from a National Science Foundation report which stated that "...quantitative thought and understanding continue to become more important for more people..."

This keystone is used in setting a new direction for curriculum and teaching that will help students learn to think, reason, solve problems and apply mathematics in real life situations. The objectives reflect mathematical knowledge that is essential for all students' educational development and employment. Rote skills learned in a meaningless way do not prepare students for the future.

The view of computation is broadened to include basic facts, mental arithmetic, estimation and calculators. With the availability of calculators and computers in the workplace, the emphasis must be on conceptual development and problem solving. The use of calculators and computers to achieve the important mathematics objectives is suggested in Educating Americans for the 21st Century.[1]

In its Agenda for Action: Recommendations for School Mathematics of the 1980s[2], the National Council of Teachers of Mathematics recommended that problem solving be the focus of school mathematics, that basic skills be defined to encompass more than computational facility and that mathematics programs take full advantage of the power of calculators and computers at all grade levels.

A later report from the National Science Foundation, The Mathematical Sciences Curriculum K-12: What is Still Fundamental and What is Not.[3] in asking recommendations for elementary and middle school mathematics, stated "A principal theme of K-8 mathematics should be the development of number sense, including the effective use and understanding of numbers in applications as well in other mathematical contexts."[3]

The changes suggested in the NSF report would "replace excess drill in formal paper-and-pencil computations with various procedures to develop better number sense on the part of the student."

Substantially less emphasis was to be placed on paper and pencil execution of the arithmetic operations and de-emphasis was suggested for drill with larger numbers and fractions with larger denominators.

The following content recommendations were included in the NSF report:

1) Mastery of basic number facts
2) Selective use of calculators and computers
3) Mental arithmetic, estimation and approximation
4) Problem solving, including the use of calculators and computers as tools
5) Elementary data analysis, statistics and probability

6) Place value, decimals, percent and scientific notation
7) Intuitive geometry, including formulas for perimeter, area and volume
8) Algebraic symbolism and techniques, particularly in grades seven and eight.

Over the past two decades there has been an explosion of research on mathematics learning and teaching. Syntheses of some of the research can be found in companion publications, Research within Reach: Elementary School Mathematics and Research within Reach: Secondary School Mathematics.[4] Among the major national data sources on mathematics learning and mathematics teaching are those reported in the National Assessment of Education Progress[5] and the Second International Mathematics Study.[6]

There is unanimity from all reports and research results that conceptualization of mathematics and understanding of problems should be valued more highly than just correct solutions to routine exercises. Yet there is extensive documentation that students are now lacking both conceptual understanding and problem solving skill. For example, only half of the students finishing eighth grade could give the meaning of the decimal 0.52 and only 20% of them could estimate the answer to 3.04 X 5.3 as 16 given the choices of 1.6, 16, 160 and 1600. For too long, mathematics instruction has been thought of as presenting rules and providing enough drill for students to master the skill. As Simon pointed out, to survive in mathematics courses, many students attempt to compensate for their lack of understanding by memorizing mathematical procedures and formulas .

The mathematics objectives have been written to a framework of eight content strands and six process strands. The content strands are (1) whole numbers and numeration, (2) fractions, decimals, ratio and percent, (3) measurement, (4) geometry, (5) statistics and probability, (6) algebraic ideas, (7) problem solving and logical reasoning, and (8) calculators. The process strands are (1) conceptualization, (2) mental arithmetic, (3) estimation, (4) computation, (5) problem solving, and (6) calculators and computers. Many of the objectives from previous documents have been retained but enhanced with a strong focus on mathematical thinking and higher order skills. The chart that follows illustrates the relational nature of the content strands to the process stands and constitutes the framework of the mathematics objectives.

Framework:
Michigan Mathematics Objectives

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Problem Solving and Applications	Calculators and Computers
	Mathematical Processes					
Whole Numbers and Numeration						
Fractions, Decimals, Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving and Logical Reasoning						
Calculators						

New Directions

The following seven topics illustrate the thrust established by the broadened framework of content strands and mathematical processes.

1. Conceptualization of mathematical topics receives much greater emphasis. Conceptual understandings are essential for all content strands, are the basis for the teaching of computational topics and are essential for success in problem solving. The increased emphasis on quantitative thinking and conceptual knowledge requires a concomitant increase in the use of manipulative materials at all grade levels, K-12.

2. Problem solving permeates all content strands. Because of their central purpose in mathematics and in practical situations, problem solving and logical thought are viewed as threads that run through all content areas. Beyond the use of problem solving in the other content areas, there are problem solving strategies in a separate strand that can help students be better problem solvers.

3. Graphical representation and interpretation are strengthened. The topics are important in mathematics and just as important in social science and science. The importance of the topic for the ordinary citizen is further underscored by the extensive use of graphs to present quantitative information in newspapers and magazines.

4. The needs of students in a technological society are reflected. The objectives take into account the major shift in the use of calculators and computers to do routine computations by the ordinary citizen and by people in business and industry. Paper/pencil computation remains as an expectation but with limits on the expectations. The objectives reflect the value of the calculator as a teaching tool and as a tool for students in solving problems beginning in the early grades. Capstone objectives for grades 4, 7 and 10 specify objectives which should be met without a calculator and those for which a student may use a calculator.

5. Mental arithmetic and estimation receive more attention and are given greater importance. One major reason for this is at least three-fourths of the everyday use of mathematics is without paper and pencil. Success depends upon sound concepts of numbers and continuing emphasis on thinking skills.

6. Spatial visualization and geometry are broadened. This shift in emphasis in the content reflects the needs of the

ordinary citizen as she/he lives in a three-dimensional world as well as the needs in later mathematics courses. The increased attention to conceptualization requires more concrete models with more emphasis on spatial and quantitative visualization.

7. Algebraic concepts and symbolism are introduced earlier. A major reason for this is the economy of algebraic symbolism to express concisely a generalization from arithmetic. For example, algebraic symbolism is esscntial in expressing the formulas for perimeter, area and volume. Further, with content from the first year of algebra becoming increasingly essential for future career choices of all students, a better transition from arithmetic to algebra is essential. Earlier use of algebraic symbolism in a meaningful way will help much in the transition from arithmetic to the more formal work in first year algebra.

These new directions for the mathematics objectives augment the basic computational skills by giving greater attention to thinking skills and reasoning. It is the thinking and reasoning person who will find a firm place in the twenty-first century. The upgrading of benchmark expectations is an important component of this forward look.

The Objectives

Objectives have been written for each of the eight content strands reflecting the five mathematical processes, as shown graphically in the previous chart. A few global objectives are given for each content strand to provide an overview of the scope of the objectives. Specific objectives with grade level expectations are provided where appropriate under the mathematical processes headings - conceptualization, mental arithemetic, estimation, computation, problem solving, calculators and computers. Not all process headings were appropriate for some global objectives.

The content strands problem solving and logical reasoning and calculators are unique because they encompass both content and process. Each of the other six content strands gives specific attention to problem solving and calculators/computers in that strand. The problem solving and logical reasoning strand concentrates on the problem solving process — strategies and logical reasoning. The calculator strand contains specific objectives relative to calculator keys and special functions on a calculator.

WHOLE NUMBERS AND NUMERATION

NUMERATION

To read, write, compare, order and round numbers.	K-3	4-6	7-9
CONCEPTUALIZATION			
To translate among models, word names and symbols.	◊		
To read numbers and recognize place value.	◊	◊	
COMPUTATION			
To compare and order numbers.	◊	◊	
To regroup numbers using place value, as needed for computation algorithms.	◊	◊	
ESTIMATION			
To round numbers.	◊	◊	

ADDITION

To add whole numbers using manipulative models and computational algorithms.	K-3	4-6	7-9
CONCEPTUALIZATION			
To identify and use models and thinking strategies for basic facts.	◊		
To identify models to show addition of multiples of 10 and 100.	◊		
To use models to show the addition algorithm, identifying results of regrouping.	◊		
COMPUTATION			
To recall basic facts from memory.	◊		
To add two numbers.	◊		
To add three or more numbers.	◊	◊	

MENTAL ARITHMETIC

To add multiples of ten or multiples of 100 mentally.	◊		
To add two two-digit numbers mentally.		◊	

ESTIMATION

To estimate the sum of two, three, or more numbers.		◊	◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving addition.	◊	◊	

CALCULATORS

To add any numbers in column or horizontal form.	◊	◊	

SUBTRACTION

To subtract whole numbers using manipulative models and computational algorithms.	K-3	4-6	7-9

CONCEPTUALIZATION

To identify and use take away and comparison models for subtraction.	◊		
To identify models to show subtraction of multiples of 10 and 100.	◊		
To relate subtraction to addition.	◊	◊	
To use thinking strategies for basic facts.	◊		
To use models to show the subtraction algorithm, identifying results of regrouping.	◊	◊	

COMPUTATION

To recall basic facts.	◊		
To find differences of two and three-digit numbers involving regrouping.	◊	◊	

MENTAL ARITHMETIC

To find differences of multiples of 10 and 100 mentally.	◊		
To find differences of two-digit numbers mentally.		◊	

ESTIMATION

To estimate to find approximate differences. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving subtraction. ◊ ◊

CALCULATORS

To subtract any whole numbers. ◊

MULTIPLICATION

To multiply numbers using manipulative models and computational algorithms. K-3 4-6 7-9

CONCEPTUALIZATION

To identify and use models and thinking strategies for basic facts. ◊

To use models to show the multiplication algorithm. ◊

MENTAL ARITHMETIC

To recall selected basic facts from memory. ◊

To find products of multiples of 10 and 100. ◊

To multiply one-digit and two-digit numbers and find other appropriate special products mentally. ◊ ◊

ESTIMATION

To use multiples of 10 and 100 to estimate products. ◊

COMPUTATION

To multiply two numbers up to a two-digit by a three-digit number. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving multiplication. ◊ ◊

CALCULATORS

To multiply any numbers. ◊

DIVISION

To divide whole numbers using manipulative models and computational algorithms.	K-3	4-6	7-9
CONCEPTUALIZATION			
To identify and use measurement and partition models for division.	◊		
To relate division to multiplication.	◊	◊	
To use thinking strategies for finding basic facts.	◊		
To interpret the remainder.		◊	
To relate models to the division algorithm.		◊	
MENTAL ARITHMETIC			
To divide multiples of 10, 100, and 1000 getting quotients that are multiples of 10, 100, or 1000.		◊	
ESTIMATION			
To use multiples of 10, 100, and 1000 to determine the number of places in the quotient.		◊	
To determine the first digit and its place value in the quotient.		◊	
COMPUTATION			
To find the quotient and remainder for one and two-digit divisors (up to 30, multiples of 10, 40 through 90) with up to four-digit dividends.		◊	
PROBLEM SOLVING AND APPLICATIONS			
To solve problems involving division.	◊	◊	
CALCULATORS			
To divide any numbers.		◊	

NUMBER PROPERTIES

To recognize and use properties of whole numbers.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To demonstrate and use the meaning of: odd and even,	◊	◊	
multiple and common multiple,		◊	
factor and common factor,		◊	
prime number and prime factorization,		◊	◊
and scientific notation.			◊
COMPUTATION			
To determine when sums, differences, products, and quotients are even or odd.		◊	
To find multiples of numbers less than 20.		◊	
To find factors of numbers less than 200.		◊	
To find common factors of two numbers, each less than 50.		◊	
To find common multiples of two numbers, each less than 16.		◊	
To determine prime numbers less than 100.		◊	
To find the prime factorization of numbers 100 or less.		◊	
To express whole numbers in scientific notation, and conversely.			◊

FRACTIONS, DECIMALS, RATIO AND PERCENT

FRACTIONS

(Note: Fractions include mixed numbers and whole numbers where appropriate.)

MEANING

To demonstrate and use the meaning of fractions.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To relate fractions to concrete models.	◊	◊	◊
To relate fractions to division using the necessary vocabulary.		◊	◊
ESTIMATION			
To estimate fractions and sizes of regions using easily recognized fractions.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems involving the meaning of fractions.	◊	◊	◊

EQUIVALENT FRACTIONS

To find equivalent fractions using concrete models and generalizations for equivalent fractions.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To relate concrete models and equivalent fractions.		◊	◊
MENTAL ARITHMETIC			
To find equivalent fractions for easily recognized fractions.		◊	◊
ESTIMATION			
To estimate fractions using easily recognized fractions.		◊	◊

COMPUTATION			
To find equivalent fractions and mixed number/fraction equivalents.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems with equivalent fractions.		◊	◊
COMPARE/ORDER			
To compare and order fractions.	K-3	4-6	7-9
CONCEPTUALIZATION			
To compare and order using models and appropriate fractions.	◊	◊	◊
ESTIMATION			
To estimate fractions using easily recognized fractions.		◊	◊
COMPUTATION			
To compare and order fractions.		◊	◊
CALCULATORS			
To compare and order fractions using decimal equivalents.			◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems involving comparing or ordering fractions.		◊	◊
ADD/SUBTRACT			
To add and subtract fractions including combinations with whole numbers.	K-3	4-6	7-9
CONCEPTUALIZATION			
To relate the addition and subtraction operations to models and to each other.		◊	◊
MENTAL ARITHMETIC			
To find sums or differences of like fractions mentally.		◊	◊
ESTIMATION			
To estimate sums and differences.		◊	◊

COMPUTATION

To find sums or differences. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving addition and subtraction with fractions. ◊ ◊

MULTIPLY/DIVIDE

To multiply and divide fractions including combinations with whole numbers.. K-3 4-6 7-9

CONCEPTUALIZATION

To relate the multiplication and division operations to models and to each other. ◊ ◊

MENTAL ARITHMETIC

To find a fractional part of appropriate whole numbers mentally. ◊ ◊

ESTIMATION

To estimate products and quotients. ◊ ◊

COMPUTATION

To find products and quotients. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving multiplication and division with fractions. ◊ ◊

DECIMALS

MEANING

To demonstrate and use the meaning of decimals. K-3 4-6 7-9

CONCEPTUALIZATION

To relate decimals to models. ◊ ◊ ◊

To use place value and to read and write decimals to thousandths. ◊ ◊

ESTIMATION

To estimate decimals using whole numbers and models. ◊ ◊

To round decimals to a given place. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving the meaning of decimals. ◊ ◊

EQUIVALENT DECIMALS

To find equivalent decimals using models and generalizations for equivalent decimals. K-3 4-6 7-9

CONCEPTUALIZATION

To identify equivalent decimals using models and generalizations for equivalent decimals. ◊ ◊

ESTIMATION

To use equivalent decimals to make estimates using models or using decimals. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems with equivalent decimals. ◊ ◊

CALCULATORS

To interpret calculator displays for decimal equivalents. ◊ ◊

COMPARE/ORDER

To compare and order decimals. K-3 4-6 7-9

CONCEPTUALIZATION

To compare or order decimals using concrete models, word names, or decimal symbols. ◊ ◊

ESTIMATION

To estimate decimals using easily recognized fractions. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving comparing or ordering of decimals. ◊ ◊

ADD/SUBTRACT

To add and subtract decimals.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To relate the addition and subtraction operations to models and to each other.		◊	◊
MENTAL ARITHMETIC			
To add and subtract selected decimals mentally.		◊	◊
ESTIMATION			
To estimate sums and differences.		◊	◊
COMPUTATION			
To add and subtract decimals.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems involving addition and subtraction of decimals.		◊	◊
CALCULATORS			
To add and subtract decimals.		◊	◊

MULTIPLY/DIVIDE

To multiply and divide decimals.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To relate the multiplication and division operations to models and to each other.		◊	◊
To relate equivalent expressions for the operations, including multiplication of a whole number and a decimal.		◊	◊
MENTAL ARITHMETIC			
To multiply and divide with decimals and powers of ten.		◊	◊
ESTIMATION			
To estimate products and quotients.		◊	◊

COMPUTATION

To multiply and divide decimals up to thousandths. ◊

CALCULATORS

To find products and quotients. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving multiplication and division of decimals. ◊ ◊

RATIO AND PROPORTION

RATIO

To use ratio in practical situations. K-3 4-6 7-9

CONCEPTUALIZATION

To determine ratios from models that are part-to-part, part-to-whole, or rates and recognize verbal expressions for ratio. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving ratios. ◊ ◊

EQUILVALENT RATIOS/PROPORTION

To identify and find equivalent ratios. K-3 4-6 7-9

CONCEPTUALIZATION

To demonstrate the meaning of equivalent ratios using models or practical situations. ◊

COMPUTATION

To find equivalent ratios and solve proportions. ◊

PROBLEM SOLVING AND APPLICATIONS

To solve proportion problems. ◊

CALCULATORS

To solve proportions with larger numbers and proportion problems with more difficult computation. ◊

PERCENT

MEANING

To demonstrate the meaning of percent as a ratio whose second term is 100.

K-3	4-6	7-9

CONCEPTUALIZATION

To use models to represent percents.	◊	◊

PROBLEM SOLVING AND APPLICATIONS

To use the meaning of percent in solving practical problems.	◊	◊

PERCENT, FRACTION, DECIMAL EQUIVALENTS

To express ratios as percents, fractions, or decimals and to relate each form to the other two.

K-3	4-6	7-9

CONCEPTUALIZATION

To recognize equivalent expressions involving selected fractions, decimals and percents using models or easily recognized fractions.	◊	◊

MENTAL ARITHMETIC

To use easily recognized fractions and give fraction, decimal and percent equivalents.		◊

ESTIMATION

To estimate equivalents for fractions, decimals and percent using easily recognized fractions.		◊

CALCULATORS

To express any ratio as a percent or decimal.	◊	◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems using fraction, percent and decimal equivalents.	◊	◊

USING PERCENT

To find a percent of a number.	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize and use the meaning of percent in finding either the part (percentage) or the whole (base) when the percent (rate) is given.		◊	◊
MENTAL ARITHMETIC			
To find selected percents of a number mentally:			
a. 1%, 10%, 50%, 100%			◊
b. 200%, 300% and other multiples of 100%			◊
c. 5%, 15%, 20%, 25%			◊
ESTIMATION			
To estimate the percent of a number using easily recognized fractions.			◊
CALCULATORS			
To find a percent of a number.			◊
PROBLEM SOLVING AND APPLICATIONS			
To solve percent problems, including percent of increase or decrease.			◊

MEASUREMENT

LENGTH, AREA, VOLUME, ANGLES

To measure length, area, volume and angles.	K-3	4-6	7-9
CONCEPTUALIZATION			
To identify and describe the concept of length and the relative sizes of the standard units.	◊	◊	
To identify and describe concepts of area, perimeter, volume and angle measure.		◊	◊
To distinguish among situations which call for measuring length, area or volume.		◊	◊
To identify and describe concepts of circumference and surface area.			◊
To determine the length of an object or a line segment with an appropriate unit and a standard measuring instrument using hands-on activities.	◊	◊	
To measure area (square units) and volume (cubic units) by the process of covering, filling, and counting and to recognize the relative size of standard units.		◊	◊
To measure a given angle and to draw an angle of a given size.			◊
To read various scales such as rulers and protractors.	◊	◊	◊
ESTIMATION			
To estimate the length of a familiar object or drawing.	◊	◊	
To estimate the area or volume of a familiar object or drawing.		◊	◊
To estimate length, area and volume using all appropriate units of measure.			◊
PROBLEM SOLVING AND APPLICATIONS			
To determine the perimeter of an object or of a polygon.		◊	

To use the formula, A = L X W, to find the area of a rectangular object or drawing.		◊	
To determine the circumference of a circle, the area of a geometric shape, and the volume of a cylinder or rectangular prism.		◊	
To use a formula to relate lengths, area, and volumes. (For example, to find the effect on the area or volume of an object by changing one dimension.)			◊
To find the area and volume of figures resulting from combining or separating common geometric figures.			◊

CAPACITY, MASS, TIME, TEMPERATURE,

To measure and use liquid capacity, mass (weight), time, temperature, monetary value and relationships of the basic metric units.	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize and use the concepts of mass, liquid capacity, time and temperature, including standard units, relative sizes, comparisons and their abbreviations and symbols.	◊	◊	
To tell time to the nearest five minutes.	◊		
To measure liquid capacity and mass (weight) using appropriate standard units and measuring instruments.	◊	◊	
To recognize and use U.S. coins and bills, $5 and less.	◊		
To read various scales, such as a thermometer.	◊	◊	
To recognize and use the characteristics of the measurement process, including selection of appropriate units, derived units, the role of approximation and the conversion-of-units process.		◊	◊
To recognize and use the metric system, including the decimal relationship among the various units and the relationships among cubic units, capacity units and mass units.			◊

To recognize and use the concept of precision of measurement.			◊
ESTIMATION			
To make estimations involving temperature, time and money.	◊	◊	
To make estimations of the capacity of various common containers in terms of metric units.		◊	◊
To make estimations of weight in terms of metric units.			◊
PROBLEM SOLVING AND APPLICATIONS			
To solve one-step verbal arithmetic problems, posed within a measurement context, including elapsed time and money.	◊	◊	
To use a table of equivalents to solve simple problems involving the conversion of units within a system of measurement.		◊	◊
To solve multi-step verbal problems posed within a measurement context.		◊	◊

GEOMETRY

SHAPE

To recognize and use shapes in one, two and three dimensions. K-3 4-6 7-9

CONCEPTUALIZATION

To identify and illustrate appropriate geometric shapes. ◊ ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems involving appropriate geometric shapes. ◊ ◊ ◊

SHAPE PROPERTIES

To recognize and use properties of one, two and three dimensional shapes such as equal sides, equal angles and symmetry. K-3 4-6 7-9

CONCEPTUALIZATION

To identify or illustrate properties of appropriate geometric shapes. ◊ ◊ ◊

ESTIMATION

To compare visually the measures (sizes) of segments, angles and plane regions. ◊ ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems using properties of appropriate geometric shapes. ◊ ◊ ◊

RELATIONS AMONG GEOMETRIC OBJECTS

To recognize and use the relations of congruence, similarity, intersection, parallelism and perpendicularity for appropriate figures in one, two and three dimensions. K-3 4-6 7-9

CONCEPTUALIZATION

To identify and illustrate appropriate relations among figures. ◊ ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems using the appropriate relations among shapes.	◊	◊	◊

POSITION

To recognize and use informal and formal coordinate systems on lines and planes to specify locations and distances.	K-3	4-6	7-9

CONCEPTUALIZATION

To identify and produce points satisfying given conditions.	◊	◊	◊

ESTIMATION

To estimate distances and positions in the coordinate plane.	◊	◊	◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems using position concepts and notation.	◊	◊	◊

TRANSFORMATIONS

To recognize and use the transformations of reflection in a line (flip), translation (slide), rotation about a point (turn), and size change (enlargement and reduction).	K-3	4-6	7-9

CONCEPTUALIZATION

To recognize and produce appropriate transformations.		◊	◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems using appropriate transformations.		◊	◊

VISUALIZING-SKETCHING-CONSTRUCTING

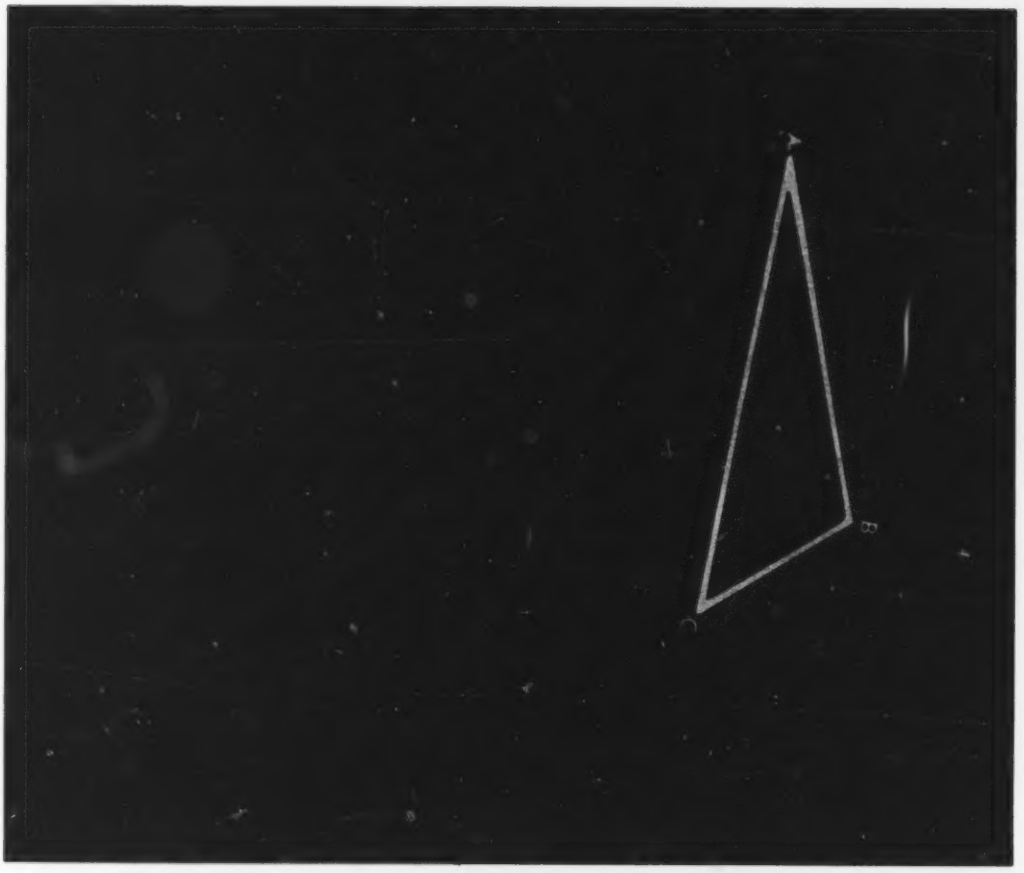
To visualize, sketch and construct geometric objects.	K-3	4-6	7-9

CONCEPTUALIZATION

To visualize, sketch and construct geometric shapes or relationships.	◊	◊	◊

PROBLEM SOLVING AND APPLICATIONS

To solve problems requiring visualizing, sketching or constructing geometric shapes or relationships. ◊ ◊ ◊



STATISTICS AND PROBABILITY

TABLES

To construct, read and interpret tables.	K-3	4-6	7-9
CONCEPTUALIZATION			
To read tables and identify existing patterns in tables.	◊	◊	
COMPUTATION			
To construct tables from data.	◊	◊	
To record data in existing tables.	◊	◊	
PROBLEM SOLVING AND APPLICATIONS			
To use tables for comparisons.		◊	
CALCULATORS AND COMPUTERS			
To generate tables using calculators and computers.		◊	◊

GRAPHS

To construct, read and interpret graphs.	K-3	4-6	7-9
CONCEPTUALIZATION			
To read graphs:			
Picture graphs, Bar graphs.	◊	◊	◊
Line graphs, Line plots.		◊	◊
Circle graphs.			◊
Stem-and-leaf plots, Box plots and Scatter plots.			◊
ESTIMATION			
To make comparisons among graphs.		◊	◊
To interpolate on graphs.		◊	◊
To extrapolate on graphs.			◊
To use a fitted line on a scatter plot for prediction.			◊

COMPUTATION

To determine appropriate scales for graphs.		◊	◊
To construct graphs.	◊	◊	◊

PROBLEM SOLVING AND APPLICATIONS

To select a graph that fits given information.	◊		
To determine patterns, see trends, predict outcomes and make wise choices using graphs.		◊	◊

DESCRIPTIVE STATISTICS

To read, interpret, determine and use descriptive statistics.	K-3	4-6	7-9

CONCEPTUALIZATION

To define terms:

Mean, median, range, frequency.		◊	
Outlier, quartile.			◊
To explain how extreme values affect the median and mean.			◊

COMPUTATION

To order data in ascending or descending order.		◊	
To determine mean, median and range.		◊	◊
To determine outlier and quartile.			◊

PROBLEM SOLVING AND APPLICATIONS

To determine patterns, see trends, predict outcomes and make wise choices using descriptive statistics.		◊	◊

PROBABILITY

To read, interpret, determine and use probabilities.	K-3	4-6	7-9

CONCEPTUALIZATION

To compare the likelihood of simple events.	◊	◊	
To designate events as certain or impossible.			◊
To give one as the sum of the probabilities of all possible outcomes.			◊

MENTAL ARITHMETIC		
To determine probabilities of simple events.	◊	◊
To determine the probability an event will not occur, given the probability the event will occur.		◊
COMPUTATION		
To determine probabilities of compound events.		◊
PROBLEM SOLVING AND APPLICATIONS		
To use probability devices to simulate real world events.	◊	◊
CALCULATORS AND COMPUTERS		
To use calculators to determine probabilities.		◊
To use computers to simulate compound events.		◊

ALGEBRAIC IDEAS

VARIABLES

EXPRESSIONS

To understand and use expressions containing variables.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize and use the concept of variable in expressions.	◊	◊	◊
COMPUTATION			
To evaluate expressions.		◊	◊
ESTIMATION			
To estimate values of expressions.		◊	◊
CALCULATORS			
To use calculators to evaluate expressions.			◊

VERBAL, SYMBOL, MODEL RELATIONS

To use variables in translating among verbal expressions, symbols, and situations that are pictorial or practical.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize physical or pictorial models for relations and operations.	◊	◊	
PROBLEM SOLVING AND APPLICATIONS			
To solve problems represented physically, pictorially, symbolically or verbally.		◊	◊

OPEN SENTENCES

To use variables to write and solve open sentences.

	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize and use the concept of variable in open sentences.	◊	◊	◊

COMPUTATION

To find solutions to open sentences. ◊ ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To find solutions to problems stated verbally. ◊ ◊ ◊

REAL NUMBERS AND PROPERTIES

DISTRIBUTIVE PROPERTY

To recognize and apply the distributive property. K-3 4-6 7-9

CONCEPTUALIZATION

To recognize equivalent manipulative or pictorial representations of the distributive property. ◊ ◊

MENTAL ARITHMETIC

To use the distributive property for mental arithmetic short cuts. ◊ ◊

PROBLEM SOLVING AND APPLICATIONS

To apply the distributive property to problem solving situations. ◊ ◊

INTEGERS

To recognize, use and compute with integers. K-3 4-6 7-9

CONCEPTUALIZATION

To interpret and compare integers in familiar situations. ◊ ◊

COMPUTATION

To determine the sign of the answer for integer computation. ◊

To compute with integers. ◊

PROBLEM SOLVING AND APPLICATIONS

To use integers in everyday situations. ◊ ◊

EXPONENTS, POWERS AND ROOTS

To recognize and use concepts of exponents, powers and roots.	K-3	4-6	7-9
CONCEPTUALIZATION			
To recognize and use patterns of squares and cubes.	◊	◊	
To recognize and use exponents and power notation.		◊	◊
To read graphs of powers and roots.			◊
ESTIMATION			
To estimate square roots.		◊	◊
CALCULATORS			
To use calculators to find or approximate solutions to exponential equations.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems involving powers and roots.			◊

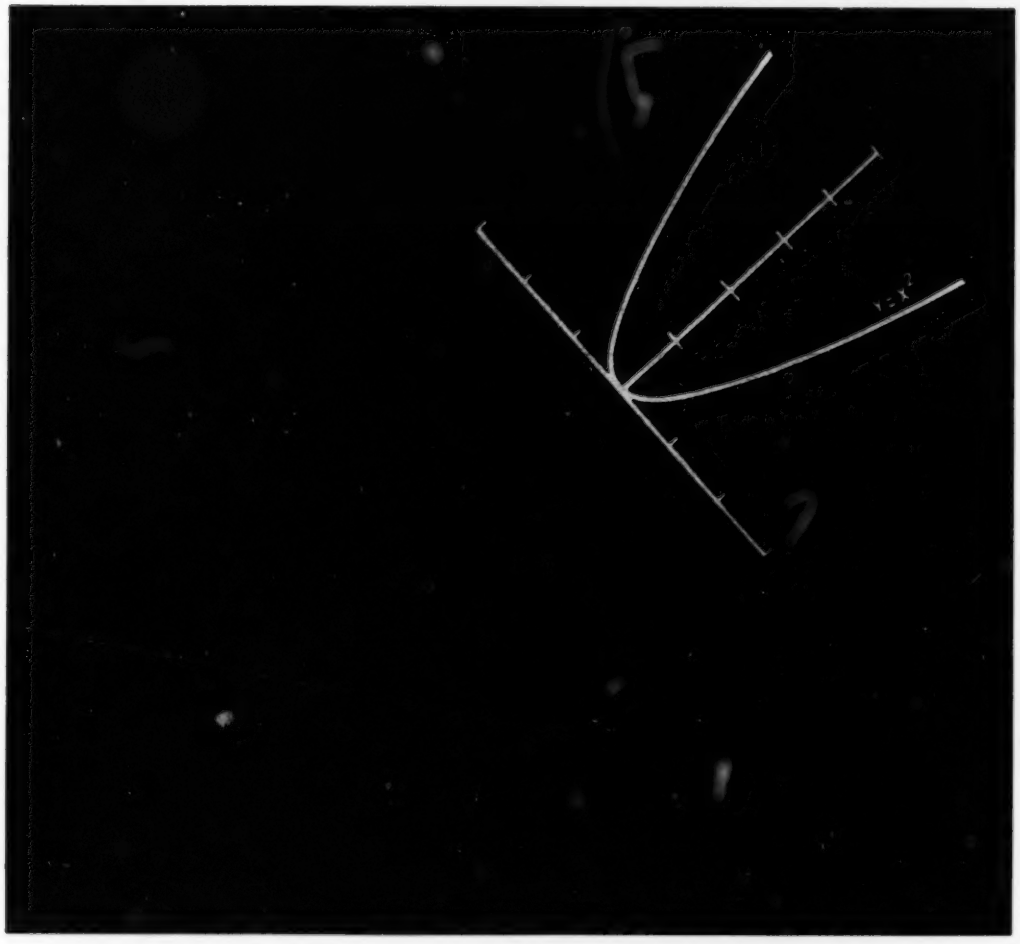
FUNCTIONS AND GRAPHS

FUNCTIONS

To recognize and use function concepts.	K-3	4-6	7-9
COMPUTATION			
To represent a function with a table of values or a graph.		◊	◊
To recognize, describe, and express in symbols a relationship between two sets.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To solve problems using functions.		◊	◊

GRAPHS

To identify and interpret graphs representing situations, tables of values or sentences.	K-3	4-6	7-9
CONCEPTUALIZATION			
To identify an appropriate graph given a table of values or an equation and conversely.		◊	◊
PROBLEM SOLVING AND APPLICATIONS			
To use graphs to solve problems.			◊



PROBLEM SOLVING AND LOGICAL REASONING

PATTERNS

To identify, use, and construct patterns.	K-3	4-6	7-9
To identify a pattern and determine a missing element.	◊	◊	◊
To create a pattern, given a formal rule.		◊	◊
To extrapolate by developing a formal rule for a pattern.		◊	◊

UNDERSTANDING PROBLEMS

To demonstrate an understanding of a problem.	K-3	4-6	7-9
To determine what is to be found.	◊	◊	◊
To identify necessary information to solve a problem.	◊	◊	◊
To determine insufficient information.		◊	◊
To formulate appropriate questions.	◊	◊	◊
To formulate a problem for mathematical expressions or number sentences.	◊	◊	◊

PROBLEM SOLVING STRATEGIES

To select and apply appropriate problem solving strategies.	K-3	4-6	7-9
To identify and use a pattern to solve a problem.	◊	◊	◊
To make an organized list or table to solve a problem.	◊	◊	◊
To guess and test to solve a problem.	◊	◊	◊
To work backwards to solve a problem.			◊
To make or use a drawing, a graph or a physical model to solve a problem.	◊	◊	◊

To write an open sentence to solve a problem.	◊	◊	◊
To solve a simpler problem to solve a problem.		◊	◊
To eliminate possibilities to solve a problem.	◊	◊	◊
To select the appropriate operations(s) to solve a one-step or multi-step problem.	◊	◊	◊

EVALUATING SOLUTIONS

To interpret and evaluate the solution to a problem.	K-3	4-6	7-9
To check the solution(s) with the conditions of the problem.		◊	◊
To find and evaluate alternative processes for solving the problem.		◊	◊
To formulate an extension of the problem.		◊	◊
To formulate a generalization of a given problem.			◊

LOGICAL REASONING

To use logical reasoning.	K-3	4-6	7-9
To determine the attributes used to classify a set and vice-versa.	◊	◊	◊
To interpret and use statements involving logical operations and quantifiers (and, or, not, if...then, every, all, some, no, at least, at most, each, exactly).		◊	◊
To recognize and draw valid conclusions from given information.		◊	◊

CALCULATORS

CALCULATOR KEYS AND FEATURES

To recognize specific calculator keys and selected calculator features.	K-3	4-6	7-9
To recognize specific calculator keys.	◊	◊	◊
To recognize appropriate key sequences for automatic constant features.		◊	◊
To recognize appropriate calculator keys related to selected terms associated with mathematical operations.	◊	◊	◊

COMPUTATION

To perform appropriate computations with a calculator.	K-3	4-6	7-9
To use a calculator to compute sums and differences using whole numbers.	◊		
To use a calculator to compute appropriate sums, differences, products and quotients with whole numbers, decimals and fractions.		◊	◊
To use a calculator to compute answers to percent problems including percent of increase or percent of decrease.			◊

LIMITATIONS AND CALCULATOR DISPLAY

To recognize certain common limitations to calculators and be able to interpret selected calculator-displayed symbols.	K-3	4-6	7-9
To recognize and interpret the calculator display.	◊	◊	◊
To recognize the limitations of the calculator regarding decimal numbers display and order of operations.		◊	◊

References

1. Educating Americans for the 21st Century. Washington, D.C.: The National Science Board Commission on the Pre-College Education in Mathematics, Science and Technology, 1983.

2. National Council of Teachers of Mathematics, An Agenda for Action: Recommendations for School Mathematics of the 1980's. Reston, Virginia: NCTM, 1980.

3. Conference Board of the Mathematical Sciences, The Mathematical Science Curriculum K-12: What is Still Fundamental and What is Not, Washington, D.C.: National Science Foundation, 1982.

4. Driscoll, M., Research within Reach: Elementary School Mathematics and Research within Reach: Secondary School Mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 1980 and 1982.

5. National Assessment of Educational Progress. Mathematical Knowledge and Skills: Selected Results from the Second Assessment of Mathematics, 1979; and The Third National Mathematics Assessment: Results, Trends, and Issues, Denver, Colorado: Education Commission of the States, 1983.

6. Second International Mathematics Study. Urbana, Illinois: University of Illinois, 1985.

7. Simon, M., "I Knew How to Do It in Class, But I Couldn't Do It at Home." Paper presented at the annual conference of the Association of Teachers of Mathematics of New England, Manchester, New Hampshire, 1985.

Acknowledgements

The Michigan Essential Goals and Objectives for Mathematics Education was developed cooperatively by the Michigan Department of Education and the Michigan Council of Teachers of Mathematics, which represents mathematics educators from throughout the state and includes local and intermediate district personnel and members of institutions of higher education. In 1986, the Michiagn Board of Education authorized the revision of the mathematics objectives.

Special tribute is extended to members of the mathematics revision committee for their time and effort in the formidable task writing the essential goals and objectives for mathematics education.

Michigan Council of Teachers of Mathematics Revision Committee:

Pat Huellmantel	- Flint Public Schools, Co-Chairperson
Joseph Payne	- University of Michigan, Co-Chairperson
Mary Bouck	- Portland Public Schools
Terrence Coburn	- Oakland Schools
Arthur Coxford	- University of Michigan
Robert Hage	- Troy Public Schools
Glenda Lappan	- Michigan State University
Frank Rogers	- Lansing Public Schools
Rheta Rubenstein	- Detroit Public Schools
Wayne Scott	- Genesee Intermediate School District
Albert Shulte	- Oakland Schools
Nancy Varner	- Detroit Public Schools

Many thanks are also extended to the many educators who contributed their expertise to this revision of the mathematics objectives and to Judy Mihelcic for typing each draft.

This document was prepared by:
THE MICHIGAN STATE BOARD OF EDUCATION

Bureau of Educational Services
Teressa Staten, Associate Superintendent

School Program Services
Anne Hansen, Director

Instructional Specialists Program
Sharif Shakrani, Supervisor

Graphic Designer,
Steve Light, Office of Public Affairs

Mathematics Specialist
Charles R. Allan

Typist
Darlene Bashore

For additional information contact:

Instructional Specialist Program
Michigan Department of Education
P.O. Box 30008
Lansing, Michigan 48909
Phone: (517) 373-1024

MICHIGAN STATE BOARD OF EDUCATION STATEMENT OF ASSURANCE OF COMPLIANCE WITH FEDERAL LAW

The Michigan State Board of Education complies with all Federal laws and regulations prohibiting discrimination and with all requirements and regulations of the U.S. Department of Education. It is the policy of the Michigan State Board of Education that no person on the basis of race, color, religion, national origin or ancestry, age, sex, martial status or handicap shall be discriminated against, excluded from participation in, denied the benefits of or otherwise be subject to discrimination in any program or activity for which it is responsible or for which it receives financial assistance from the U.S. Department of Education.

Framework:
Michigan Mathematics Objectives

Mathematical Processes

Mathematical Content Strands	Conceptualization	Mental Arithmetic	Estimation	Computation	Problem Solving and Applications	Calculators and Computers
Whole Numbers and Numeration						
Fractions, Decimals, Ratio and Percent						
Measurement						
Geometry						
Statistics and Probability						
Algebraic Ideas						
Problem Solving and Logical Reasoning						
Calculators						